Plug-and-play Diffusion Models for Image Compressive Sensing with Data Consistency Projection

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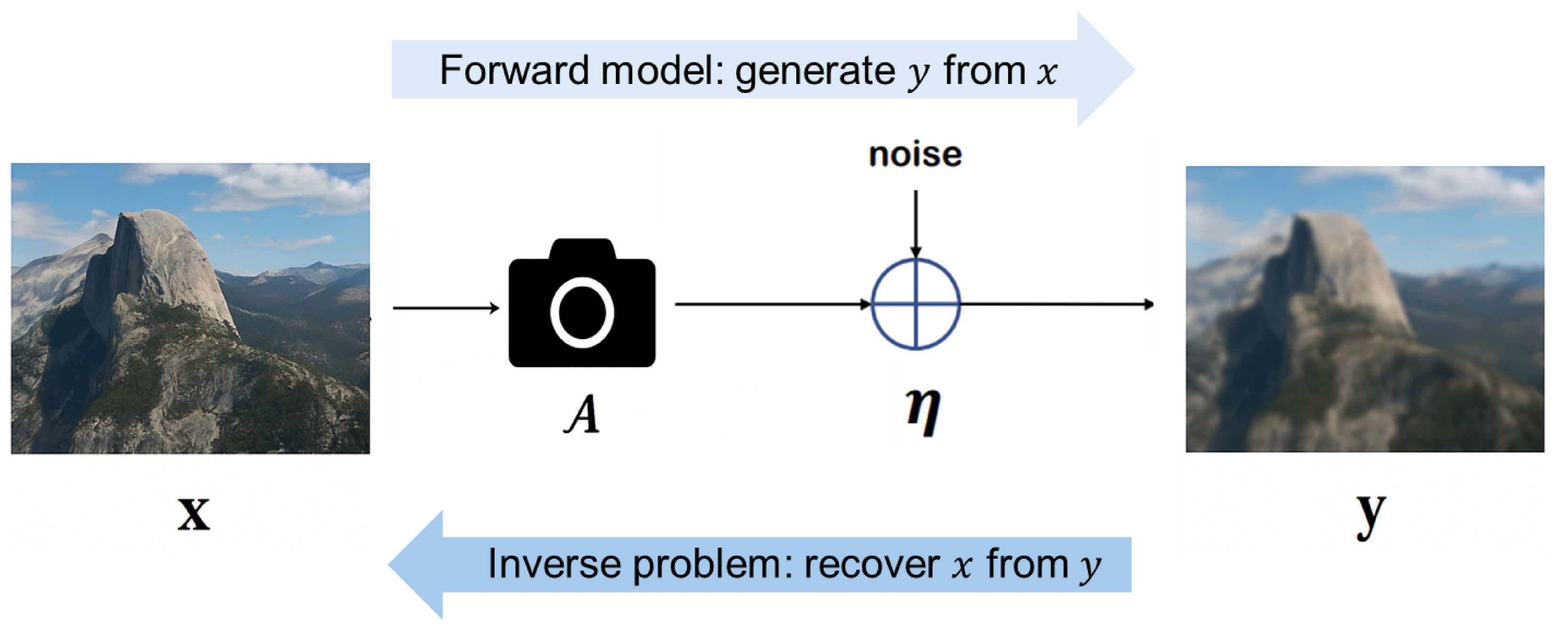
Introduction

We investigate the between plug-and-play and diffusion model for inverse problem.

- Acquiring the image x is an inverse problem.
- Solving an inverse problem is an ill-posed **one-to-many mapping** problem.



Methodology

Maximum a posteriori probability (MAP) estimator for inverse problem

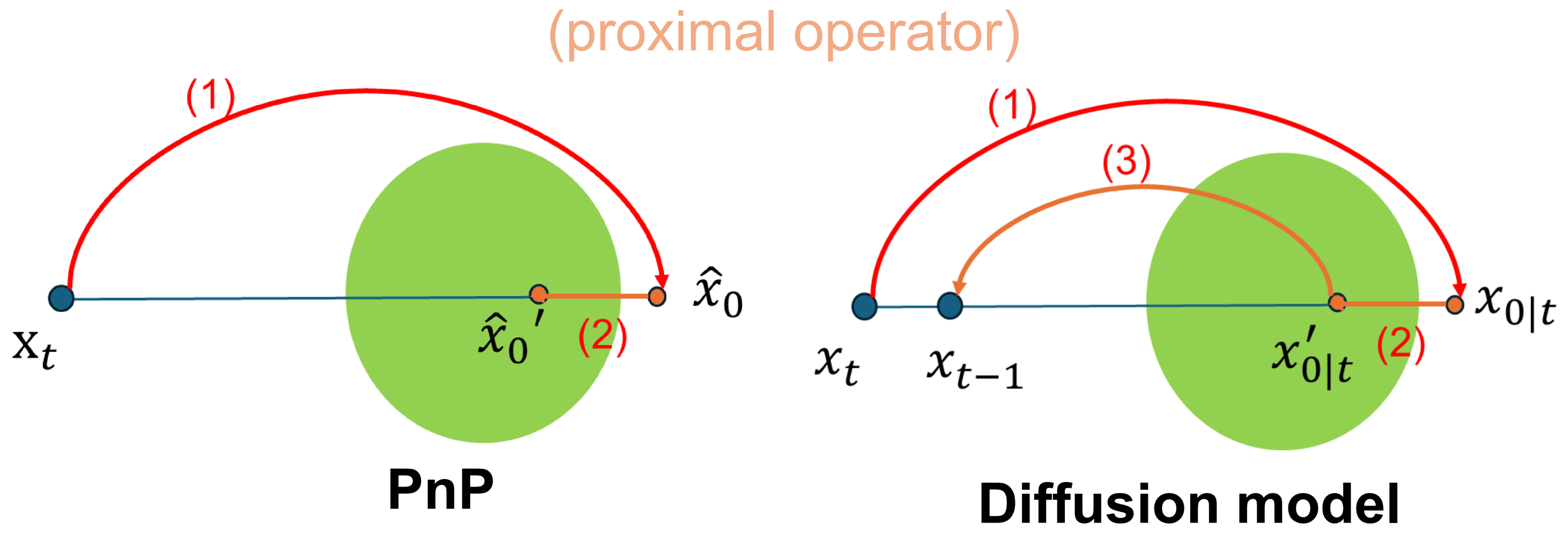
$$\hat{x} = \arg\min_{x \in \mathcal{R}^n} \frac{1}{2\sigma^2}\|y - Ax\|_2^2 + h(x), \quad h(x) = -\log(\mathcal{P}_x(x))$$

Proximal operator is an optimization-based image denoiser

$$\mathrm{prox}_\sigma(z) = \arg\min_{x \in \mathcal{R}^n} \frac{1}{2\sigma^2}\|z - x\|_2^2 + h(x)$$

$$x^t \leftarrow \mathrm{prox}_\sigma(z^t) \qquad z^t \leftarrow x^t - \gamma\, W\, \nabla_{x^t}\|y - Ax^t\|_2^2$$

Plug-and-play (PnP) & Diffusion use denoiser as prior (proximal operator)



Diffusion model

Ours: Fused update of GAP and HQS for Diffusion

Algorithm 1 Fused Data Guidance for Diffusion Sampling

Require: Observation $\mathbf{y}$, SPI forward model $\mathbf{H}$, score function, diffusion schedule $\{\sigma_t\}$, fusion weights $\{\delta_t\}$

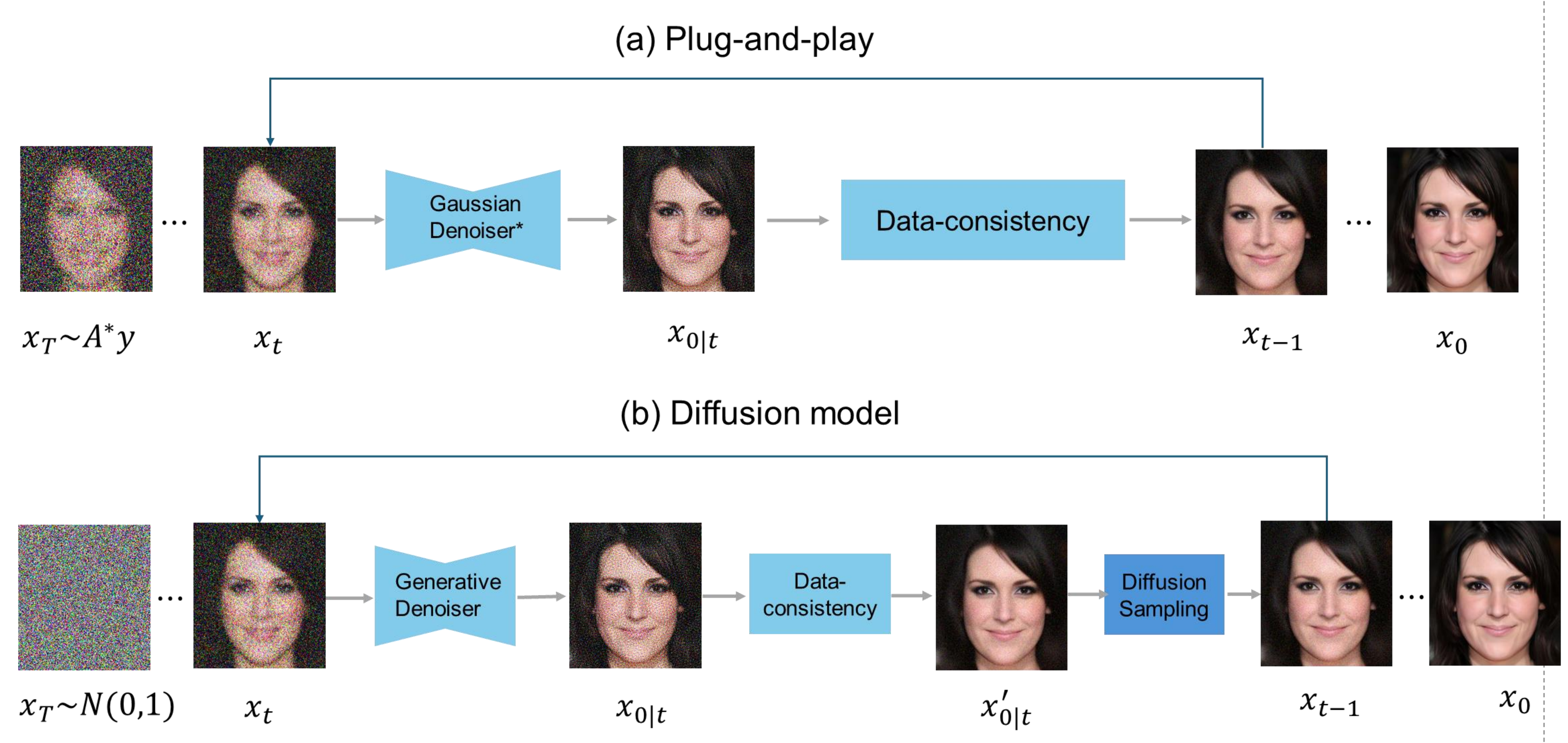
Initialize $\mathbf{x}_T \sim \mathcal{N}(0, \mathbf{I})$
for $t = T$ to 1 **do**
$\mathbf{x}_{0|t} \leftarrow \frac{\mathbf{x}_t + (1-\alpha_t)\nabla_{\mathbf{x}_t}\log p_t(\mathbf{x}_t)}{\sqrt{\alpha_t}}$ // Denoise
$\mathbf{g}_{\mathrm{GAP}} \leftarrow \mathbf{x}_{0|t} + \mathbf{H}^\dagger(\mathbf{y} - \mathbf{H}\mathbf{x}_{0|t})$ // GAP term
$\mathbf{g}_{\mathrm{HQS}} \leftarrow (\mathbf{H}^T\mathbf{H} + \lambda\mathbf{I})^{-1}(\mathbf{H}^T\mathbf{y} + \lambda\mathbf{x}_{0|t})$ // HQS term
$\mathbf{x}'_{0|t} \leftarrow (1-\delta_t)\mathbf{g}_{\mathrm{GAP}} + \delta_t \mathbf{g}_{\mathrm{HQS}}$ // Fused term
$\hat{\epsilon}_t \leftarrow \frac{(\mathbf{x}_t - \sqrt{\alpha_t}\,\mathbf{x}'_{0|t})}{\sqrt{1-\alpha_t}}$
$\epsilon_t \sim \mathcal{N}(0, \mathbf{I}_n)$
$\mathbf{x}_{t-1} = \sqrt{\bar{\alpha}_{t-1}}\,\mathbf{x}'_{0|t} + \sqrt{1-\bar{\alpha}_{t-1}}\left(w_t\sqrt{1-\zeta}\,\hat{\epsilon}_t + \sqrt{\zeta}\,\epsilon_t\right)$
// DDIM Sampling
end for
return $\mathbf{x}_0$

PnP and Diffusion model share the same data-fidelity and denoising updating.

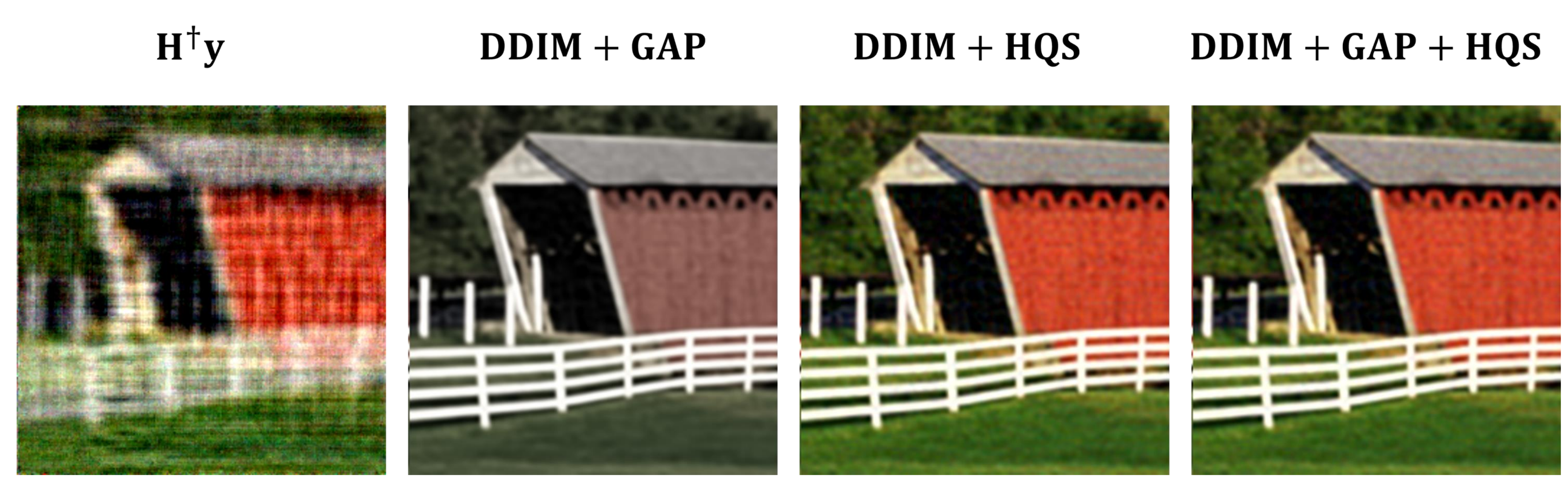
We propose a fused measurement-guidance (Generalized Alternating Projection and Half Quadratic Splitting) for data-fidelity updating.

Results

We explore the connection between PnP and diffusion model



Visual experiments when incorporating hybrid data-fidelity



Experimental Analysis

Table 1. Reconstruction metrics comparison across methods.

Method	PSNR (dB)	SSIM	LPIPS
$\mathbf{H}^\dagger\mathbf{y}$	20.55	0.39	0.54
DDIM+GAP	24.09	0.62	0.33
DDIM+HQS	24.64	0.67	0.38
DDIM+GAP+HQS	24.76	0.68	0.37

Table 2. Reconstruction Results across different CRs.

CR	PSNR	SSIM	LPIPS
1%	21.23	0.47	0.56
5%	24.76	0.68	0.37
10%	25.66	0.70	0.25
20%	27.01	0.78	0.18

Conclusions & References

We investigate the between plug-and-play and diffusion model for inverse problem.

- Plug-and-play and Diffusion model share the same iterative step in data-fidelity and denoising
- We proposed a hybrid data-fidelity guidance for diffusion model. And it has been validated effective in image compressive sensing.